WORCESTER POLYTECHNIC INSTITUTE
TWENTY-SIXTH ANNUAL INVITATIONAL MATH MEET
OCTOBER 22, 2013
TEAM EXAM QUESTION SHEET WITH ANSWERS

DIRECTIONS: Please write your answers on the **TEAM ANSWER SHEET** provided. This part of the contest is 45 minutes. All 14 problems are counted equally. Calculators and other elecrtonics **MAY NOT** be used.

1. **Solve for $x < 0$ $2\log_4(x) = \log_4(x) + \log_4(4x + 9)$**

   Solution: $x = -3$

2. **If $\log_9(\log_3(\log_2(x))) = 0$ then $x^{-1/2} = ?$**

   Solution: $x^{-1/2} = \frac{1}{\sqrt{8}} = \frac{1}{2\sqrt{2}}$

3. **A contractor found that for a certain wall it would take of his bricklayers 9 hours to complete it and the other 10 hours. From experience he found that when they worked together their combined output fell by 10 bricks per hours. Being in a hurry he put both men on the job and found that it took exactly 5 hours to get the job done. How many bricks were in the wall?**

   Solution: x = 900

4. **Consider the progression $10^{1/11}$, $10^{2/11}$, $10^{3/11}$, . . . $10^{n/11}$ What is the smallest value of n for which the product of the first n terms exceeds 100,000 ?**

   Solution: n=11

5. **A triangle has sides of 25, 15, and 20 units in length. It is inscribed in a circle. What is the radius of the circle?**

   Solution: 25/2

6. **The sum of all but one of the interior angles of a convex polygon equal 2570 degrees. What is the remaining angle?**

   Solution: 130 degrees

7. **An ellipse is drawn by fastening two ends of a string down a distance 8 inches apart. The string is 10 inches long. What is the area of the resulting ellipse?**

   Solution: 18π

8. Simplify the following sum:

$$\sum_{k=1}^{n} 2^k \binom{n}{k}$$

your answer should be in terms of n

Solution: $3^n - 1$

9. What is the smallest integer n such that $\sqrt{n} - \sqrt{n-1} < .01$?

Solution: 2501

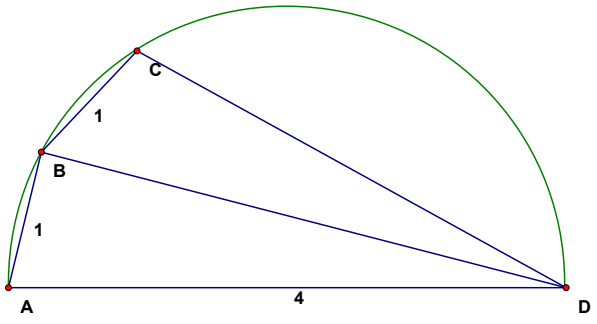
10. For positive values of x, let the function f be defined by

$$f(x) = \frac{(x+\frac{1}{x})^6 - (x^6 + \frac{1}{x^6}) - 2}{(x+\frac{1}{x})^3 + (x^3 + \frac{1}{x^3})}$$

what is the minimum value of f?

Solution: f(1)=6

11. A quadrilateral ABCD is inscribed in a circle with side AD being a diameter of length 4. If sides AB and BC each have length of 1, then side CD has what length?



Solution: CD = 7/2

12. A sphere contains 2 pennies, 4 nickels and 6 dimes. Six coins are drawn without replacement. What is the probability that the value of the coins chosen is at least 50 cents?

Solution: 127/924

13. Each whole number can be written in exactly one way in base 3/2, with each coefficient being 0, 1, or 2.

If $17=a(3/2)^4 + b(3/2)^3+c(3/2)^2+d(3/2)^1+e(3/2)^0$, find the five-digit base 3/2 number abcde.

Solution: 21012

14. Nine lines parallel to the base of a triangle divide the other sides each into 10 equal segments and the area into 10 distinct parts. If the area of the largest of these parts is 38, then what is the area of the original triangle?

Solution: 200